Forecasting of Chaotic Time Series Using RBF Neural Networks Optimized By Genetic Algorithms

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Abstract: *Time series forecasting is an important tool, which is used to support the areas of planning for both individual and organizational decisions. This problem consists of forecasting future data based on past and/or present data. This paper deals with the problem of time series forecasting from a given set of input/output data. We present a hybrid approach for time series forecasting using Radial Basis Functions Neural Network (RBFNs) and Genetic Algorithms (GAs). GAs technique proposed to optimize centers c and width r of RBFN, the weights w of RBFNs optimized used traditional algorithm. This method uses an adaptive process of optimizing the RBFN parameters depending on GAs, which improve the homogenize during the process. This proposed hybrid approach improves the forecasting performance of the time series. The performance of the proposed method evaluated on examples of short-term mackey-glass time series. The results show that forecasting by RBFNs parameters is optimized using GAs to achieve better root mean square error than algorithms that optimize RBFNs parameters found by traditional algorithms.*

Keywords: *Time series forecasting, RBF neural networks, Genetic Algorithms, Hybrid Approach.*

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1. Introduction

Forecasting is not an easy task in any scientific field, it is not an end in itself, but part of a complex decision-making process, therefore it is advisable to expose forecasting techniques in the context of real situations where apply [6]. This field is widely used in many aspects of our lives and increasingly playing an important role in almost all fields of science and engineering [7]. Time series forecasting, or time series prediction, takes an existing series of data to predict future values. A time series is a set of values that can be considered as a set of certain observations taken along the system time; the goal is to observe or model existing data series to enable future unknown data values to be forecasted accurately based on previous and present values. Basically, time series forecasting can be considered as a modelling problem. The first step is establishing a mapping between inputs/outputs. Usually, the mapping is nonlinear and chaotic. After such a mapping is set up, future values are forecasted based on past and current observations [20, 33].

Several methods have been proposed for forecasting time series, such as ARIMA methodology [9, 20]. Methods depend on computational intelligence, such as immune systems [23]. More advanced method of Soft computing solutions using Neural Networks (NNs) and Support Vector Machines (SVM) and fuzzy logic techniques [13, 14, 29]. Another method is performed by combining genetic algorithms with any of the previous soft computing methods [24].

NNs models are often used for its incredible ability to forecast nonlinear systems [32]. Radial Basis Function Neural Networks (RBFNs) are one of the most popular topologies of NNs, RBFNs has best approximation and better general performance. This computational model has proved its importance in solving problems such as function approximation, classification or nonlinear time series prediction [22]. Basically an output of a RBFN provides a weighted sum as responses of each neuron in the RBFN. The Radial Basis Function (RBF) output has local characteristics and is given in terms of a center and a width. So the goal when designing a RBFN is to determine the center and width characterizing of each RBF and weight for each. However, there is no clear rule to optimize these parameters; these parameters determine the success of the training in RBFN. GAs is successfully used to optimize RBFN parameters, because of their extensive global optimization capability. Genetic Algorithms (Gas) have been more and more used in designing RBFN in various ways: Optimization of network topology, selecting a topology of a RBFN as the number of neurons; optimization of RBFN parameters. The process of combining GAs and RBFNs is shown in Figure 1.

The typical RBFNs design algorithm has two phases. In the first one, the centers and the width of the RBFs are determined, while in the second stage their weights are calculated. One important way to determine the centers and width is clustering techniques [25]. The second by calculating the weights using traditional algorithms, such as LMS, SVD, OLS [12, 15, 30]. In most of the methods that used GAs, an individual represents complete RBFN parameters, where GAs operators by a random process to optimize the topology and all parameters of RBFN [8, 17]. RBFNs are feed forward neural networks, which have the property of universal approximation and received little attention to optimize the individual parameters using GAs [17]. RBFNs are characterized by a transfer function in the hidden unit layer having radial symmetry with respect to a center [4].

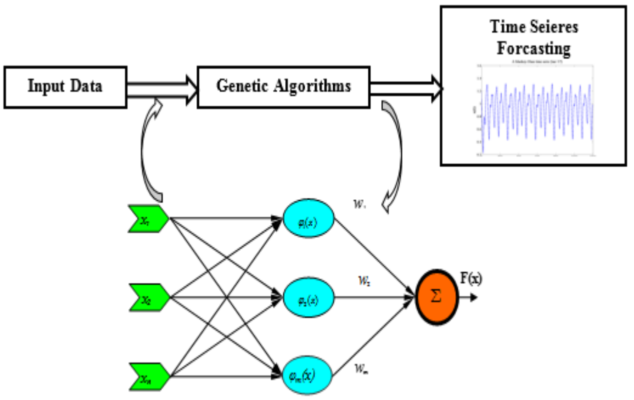

Figure 1. Combining GAs and RBFN for time Series Forecasting.

The basic architecture of RBFNs is a 3-layer network illustrated as in Figure 1. The input layer with n nodes, unique hidden layer with m RBFs and the output layer in the case of prediction has only one node. The RBFs m hidden layer activation offers a symmetrical radial $\varphi_i = R^n \rightarrow R$, which can take different forms. The most common form is the one given by a Gaussian function using the following expression:

$$\phi(\vec{x},\vec{c},r) = \exp\left(\frac{\|\vec{x}-\vec{c}\|}{r}\right) \quad (1)$$

Where $\varphi = \{\varphi_i : i = 1,...,m\}$ are the RBFs, $\vec{x}$ is the input data, $\vec{c} \in R^n$ is the center of the RBF φ, and $r \in R$ is the width of RBF. The output of the RBFNs is given by the following expression:

$$F(\vec{x}, j, w) = \sum_{i=1}^{m} j_i(\vec{x}).w_i \quad (2)$$

Where wi is the associate weights for every RBF.

Different methods exist of combining GAs and RBFN. However, most existing approaches work with typical GAs where an individual represents a complete RBFN parameter (center c, radii r and weights w) and number of RBF in the hidden layer. In the literature, the most existing approaches work with typical evolutionary methods where an individual represents a complete RBFN, this will produce problems with high computational cost and premature convergence to local minima. Hybrid evolutionary algorithms are used to automatically build a (RBF-NN) for rainfall forecasting. It uses Particle Swarm Optimization (PSO) into the GA for determining the parameters of radial basis function neural networks (number of neurons, their respective centers and radii) [31], in [27] Evolutionary Radial Basis Function algorithms (EvRBF) are used to automatically build a RBF NN that solves currency exchange rate forecasting problem. In [34] the author use GAs adopting the real coding, crossover probability and mutation probability was applied to optimize the parameters of the RBFN. In [19] research proposes a method which is RBFN hybrid with a PSO algorithm (RBF-PSO) applied on two well-known benchmark dataset Mackey-Glass Time Series (MGTS) [19]. In [3] for time series prediction, a new efficient method of clustering of the centers of RBFNN; this method uses the error of each cluster produced by real output of the RBFNN trying to concentrate more clusters in those input regions where the prediction error is bigger. In [2] the author uses new efficient approach of optimizing the position of the mother wavelet of the WNN to forecast time series; it uses the real output of WNN to move the position of wavelet single function. cooperative neuro-evolutionary methods used to employ two problem decomposition methods for training Elman recurrent neural networks on chaotic time series problems [10]. Paper [5] introduces a new hybrid learning algorithm based on an Imperialist Competitive Algorithm (ICA) for training the antecedent part and Least Square Estimation (LSE) method for optimizing the conclusion part of ANFIS. In [11] the author use particle swarm optimization algorithm to search the optimal parameters of the model. In [1] the author develops a hybrid method based on a Local Linear Neuro-Fuzzy Model (LLNF) and Optimized Singular Spectrum Analysis (OSSA), termed OSSA–LLNF. Paper [21] presents a new hybrid evolutionary algorithm for determining both architecture and parameters of RBFNNs, generates a new architecture applying Particle Swarm Optimization (PSO) is used to determine the training parameters efficiently.

In this paper, a proposed approach of combining GAs and RBFN is presented based on different way of optimizing the topology of RBFNs and its parameters (centres c, width r). RBFN weights w calculated by Orthogonal Least Squares (OLS) [12]. The initialization of the population in GAs depends on k-means clustering [18] to initialize the RBF centers and K-Nearest Neighbors technique (k-nn) to initialize RBF width [30]. Each individual is an entire set of chromosomes cooperate to constitute
RBFNs. In the proposed approach, we use the incremental method to determine the number of RBF depending on the data test error that the system produces.

The organization of the rest of this paper is as follows: Section 2 presents an overview of the proposed time series forecasting method. Section 3 presents in detail the proposed method of combining

GA and RBFN for time series forecasting. Then, Section 4 shows some results that confirm the performance of the proposed method. Some conclusions will be presented in Section 5

2. Proposed Time Series Forecasting Method

Forecasting future values of the time series depends on previous values and the present value of the time series, these values are used as input to the model, $x_{t+h} = F_h(x_t,...,x_{t-t_w}) + \varepsilon_h$, where x_{t+h} is the forecasting forward steps h with respect to time t, F_h is the modelling function between the previous and future values. t_w is the input data of the function F, ε_h the modelling error. One challenge in time series forecasting is long term time Series forecasting, which in general is more complex than short-term time series forecasting. In this work, we use Mackey-Glass time series to test the performance of the proposed method. The series is a chaotic time series, making it an ideal representation of the nonlinear oscillations of many physiological processes [34]. A hybrid GAs with minor changes in the population initialization and robust method of crossover and mutation proposed to optimize the topology and parameters of RBFN. In this method, each individual in the population represents a basis function and the entire population is responsible for the final solution. The process begins with an initial population generated using an efficient clustering algorithm to initialize the RBF centers, the K-nn technique, used for the initialization of the RBF width, orthogonal least squares (OLS) to directly optimize the weights of the RBFN. Root Mean Squared Error (RMSE) used as fitness function to evaluate the each chromosome in the population. The minimum cost chromosome will be selected for the next generation, each individual cooperate to achieve the final solution, and also compete for survival. If the work of an individual is not good, that individual will be eliminated. Where typical genetic operators like crossover and mutation applied to produce a new population. The main steps of the proposed Hybrid Methodology are presented in the following pseudo-code:

Begin

Step 1. Initialize population of RBFs parameters {*c*[k-clustering algorithm], *r* [K-nn]};
Step 2. Initialize RBFs weights *w* Using OLS.
Step 3. Evaluate each individual of RBFs.
Step 4. Select the best individual of RBFs.
Step 5. Applying operators on selected individuals of RBFs.
Step 6. Select the best RBFs.
Step 7. If satisfied stop condition → stop
Else
Step 8. Increment the number of RBFs +1.
Step 9. Return to step 1.

End

3. Hybrid Methodology for Time Series Forecasting

GAs is a search or an optimization algorithm, which invented, based on genetics and evolution. GAs improves the performance of RBFNs by selecting the best-input features, by optimizing of RBFN parameters (centers, width and number of RBF in hidden layers, weights). The objective of using GAs is to provide a methodology for search allowing to find a dynamic RBFN model that maximizes the time series forecasting by optimizing the RBFN parameters and topology. GAs works with a population of chromosomes, which are elements of a search space of higher dimensionality, its chromosomes are a string of float numbers, which represent the architecture of a RBFN using a direct encoding. It can be seen as an array of genes, several genes constitute where each gene takes a value according to a domain. The manipulation of these genes will allow us to make combinations to find new RBFN architectures. The initial population of individuals, which have digit string as the chromosome, is generated using fast traditional algorithms. The fittest, which is a measure of the improvement of the forecasting process, is calculated for each individual. Selection criteria are used to select the best chromosomes to the next generation depends on fitness value, to avoid falling into local minima still possible to find a suitable combination that minimizes the mistake; this also helped using efficient methods of crossover and mutation. Fitness function applies again on the selected chromosomes to select the best one. Comparing this selected chromosome with the threshold value, which enable each individual to be enhanced, by transforming parent's chromosomes into offspring's ones. This evolutionary process is repeated until it is accomplished with a stop condition, so at the end you have the best individual found by the GA, which has the highest adaptability, meaning the smallest error encountered. The next subsections explain the methodology of the proposed method.

3.1. Initializing RBFs on Population

The operation of determining the initial RBFs is simple. Neurons were randomly placed on the chromosome set samples. RBFs number is specified as a parameter (the size of the population, m). The center of each RBF, c_i, is initialized using efficient clustering method, k-Means Clustering, with k equal number of RBF used in the initialization. Assigning each input data point to the nearest clusters depends on Euclidean distance the process performed by the following pseudocode:

```
Begin
Initialize RBFi, i = 1,…,k, to k random x
        For all xt  in input data X
        Vi ← 1 if || x - RBFi || = minj || x - RBFj ||
Vi ← 0 otherwise
For all mi, i = 1,…,k
RBFi ← sum over t (Vi x) / sum over t (Vi )
End
```

The width, r_i, is initialized using K- nearest neighbour (Knn), this technique fixes the width of each RBF on a value equal to the mean Euclidean distance between the k nearest centers of RBF.

$$r_j = \frac{1}{p}\sum_{p}\left\|C_j - C_p\right\| \tag{3}$$

It means that each gene is constituted by a real vector representing a center and a real value representing its radii. Chromosomes have a variable length, which defined as follows:

$$\left[\{c_{1n}, r_{1n}\}, \{c_{2n}, r_{2n}\}, \ldots, \{c_{in}, r_{in}\}\right] \tag{4}$$

Finally, the weights, w_{ij}, are initialized with zero value and during the RBFN parameter optimization by GAs the proposed method uses OLS. OLS is a method used to solve systems of linear equations. It is an iterative technique that selects in each iteration column of activation matrix φ, which is the largest contributor to decreasing the model error. This method transforms the matrix columns $\vec{z}_j$ of the activation matrix G in a set of orthogonal vectors $\vec{u}_j$, according to the following expression:

$$G = U\ Q \tag{5}$$

Where U is the matrix of columns $\vec{u}_j$ and Q is an upper triangular matrix with ones on the diagonal. With the substitution of these parameters into equation (2), the optimized RBF weights will be calculated using the following equation

$$\vec{w}_{ij} = U.Q.\mathrm{F}(\vec{x},\varphi,w) \quad = U\vec{H} \tag{6}$$

Where $\vec{H} = Q\ \vec{y}$.

3.2. Evaluate each RBFs.

GA uses the objective functions of each RBFN as information to guide the search. The traditional training plan, either gradient descent algorithm or Least Square Method, not only have a slow convergence rate, but also was very likely to fall into local minimum points. An evaluation mechanism is required to calculate the value of the fitness in each chromosome. For this purpose, the proposed method use a fitness function is the error between the target output and the current output. *(Fitness = error)*. In this paper, the fitness function we are going to use is the so-called RMSE.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{600}\left(T_i - F(\vec{x},\varphi,w)\right)^2}{600}} \tag{7}$$

Where T_i is the objective output of the RBFN, $F(\vec{x},\varphi,w)$ is the real output after optimization process, n is the number of input data.

3.3. Select the Best RBFs.

The selection of the RBFs to produce the consecutive generation is an important step in GAs. The probable selection individuals are realized depending on the evaluation function (fitness) of the individuals that presents the RMSE between the objective output and actual output of RBFN, where the best individuals have more possibility to be selected for the next generation. There are different techniques for selecting individuals such as: roulette of selection, stochastic universal sampling, linear rank, exponential rank, geometric ranking, tournament, and truncation. The efficient method of the most common selection process assigns a probability of selection p_j to every individual i based on its value of fitness. A chance series of numbers N are generated and compared against the accumulated probability C_i equal the sum of P_i from j equal 1 to i of the population. The appropriate individual j is selected and copied in the new population if $C_{i-1} < U(0,1) \leq C_i$. In the proposed method, we use the normalized geometric ranking method; the evaluation function of the geometric method plans the solution with a partially ordered set. This method have the minimization and the negative reaction of the geometric method of classification, it assigns *Pi* based on the line of solution *I* when all the solutions are classified. The normalized geometric ranking method of definite classification the probability *Pi* for every individual is defined as:

$$P_i = q^{+}(1-q)^{r-1} \tag{8}$$

Where q is the probability of selecting the best individual, r is the line of the individual, where 1 is the best.

$$q = \frac{q}{1-(1-q)^{P}} \tag{9}$$

Where P is the size of the population. The normalized geometric ranking method selects only the best RBFN for the next generation, which increases the speed of convergence.

3.4. GAs operators on selected RBFs.

Crossover and the mutation operations provide the basic mechanism of search in GAs. The operators produce new solutions for the next generation of population based on solutions that exist from the population.

3.4.1. Crossover the best 2 RBFs

The crossover provides the basic mechanism of search in GAs, this operation creates new chromosome based on the selected chromosomes from the population. The proposed method uses real number coding, therefore, the corresponding intersection operation can be realized

	RBF_1			RBF_2			RBF_3			RBF_4			RBF_5		
$\tilde{X}$	c_{1X}	r_{1X}	w_{1X}	c_{2X}	r_{2X}	w_{2X}	c_{3X}	r_{3X}	w_{2X}	c_{4X}	r_{4X}	w_{4X}	c_{5X}	r_{5X}	w_{5X}
$\tilde{Y}$	c_{1Y}	r_{1Y}	w_{1Y}	c_{2Y}	r_{2Y}	w_{2Y}	c_{3Y}	r_{3Y}	w_{2Y}	c_{4Y}	r_{4Y}	w_{4Y}	c_{5Y}	r_{5Y}	w_{5Y}
$cross(X,Y)$	c_{1X}	r_{1X}	w_{1X}	c_{4Y}	r_{4Y}	w_{4Y}	c_{3X}	r_{3X}	w_{2X}	c_{4X}	r_{4X}	w_{4X}	c_{5X}	r_{5X}	w_{5X}
$cross(Y,X)$	c_{1Y}	r_{1Y}	w_{1Y}	c_{2Y}	r_{2Y}	w_{2Y}	c_{3Y}	r_{3Y}	w_{2Y}	c_{2X}	r_{2X}	w_{2X}	c_{5Y}	r_{5Y}	w_{5Y}

Figure 2. The arithmetic crossover of the best two chromosomes $\tilde{X}, \tilde{Y}$.

by arithmetic crossover; it is the cross of two RBFs parameters in the chromosomes and produces two new RBF parameters in the chromosome. Arithmetical crossover is defined as a linear combination of two individuals to generate two linear combinations of the parents (two new individuals). If parent$_1$ $\tilde{X}$ and parent$_2$ $\tilde{Y}$ are the parents with best fitness value in the population, the function returns the child's as in the following equations:

$$cross(X,Y) = \alpha\ \tilde{X} + (1-\alpha)\tilde{Y} \tag{10}$$

$$cross(Y,X) = (1-\alpha)\tilde{X} + \alpha\tilde{Y} \tag{11}$$

Where $\tilde{X}$ and $\tilde{Y}$ are the probability random value dimensional *m* (RBFs), α is probability random value between [0, 1]. Appling these equations on the selected chromosomes of RBFs parameters, we can present the process of the arithmetic crossover as shown in Figure 2 of 5 RBFs.

3.4.2. Mutation the best RBFs

The mutation process, helping the GAs mechanism preventing the search process to stack in local minima, mutation uses the current population to find the best individuals. It is also an important part in recovering the genetic information lost during the crossover operations. It changes to an individual to produce a new solution. We can find many methods of mutation. In the proposed method, we use the process of uniform mutation that changes one of the parameters of the RBF parent. The uniform mutation randomly changes the center and width of one RBF j and makes it equal to a uniform chance number inside the interval $\lfloor U(a_i, b_i) \rfloor$, is the largest integer $r \leq x$. The center and width are modified with a probability inversely proportional to the number of features in the problem. The equation that presents the uniform mutation is:

$$x_i^{'} = \begin{cases} \lfloor U(a_i, b_i) \rfloor & if\ i = j \\ x_i & otherwise \end{cases} \tag{12}$$

Where a_i and b_i are down and top level, for every variable i. Figure 3 present the process of mutation appears between the parameters of the RBFs of 5 RBFs.

After applying GAs operators appear new RBFs and these are compared with their parents to determine which have a better performance in the RBFN. The best RBFs will be chosen to be part of the new population.

3.5. Stop Condition

RBFs Parameters evolve from generation to generation selecting and reproducing parents until reaching the stop criterion. The most used criterion to stop the iterative process of GAs is pre-defined maximum generation number or if the fitness value of the current population does reach the fitness threshold. Selected chromosome with best fitness values in the current population survive and move to next population, the rest chromosomes of the next population are generated depends on some selected traditional algorithms. This process of restart GAs operation helps the proposed method not to stack in a local minimum and explore different search places. In this proposed method we use the two types; the maximum number of generations, and specified threshold, which means that the process stops if one of the two types is reached. The stop condition of the GAs process presented by the following formula:

$$Stop_c = \left| B_{Fc} - B_{Fp} \right| \leq \varepsilon\ or\ Stop_c \leq \theta \tag{13}$$

	RBF$_1$			RBF$_2$			RBF$_3$			RBF$_4$			RBF$_5$		
$cross(X,Y)$	c_{1X}	r_{1X}	w_{1X}	c_{2X}	r_{2X}	w_{2X}	c_{3X}	r_{3X}	w_{2X}	c_{4X}	r_{4X}	w_{4X}	c_{5X}	r_{5X}	w_{5X}
$x_i^{'}$	c_{1X}	r_{5X}	w_{1X}	c_{4X}	r_{4Y}	w_{4Y}	c_{3X}	r_{3X}	w_{2X}	c_{2X}	r_{4X}	w_{4X}	c_{5X}	r_{1X}	w_{5X}

Figure 3. The uniform mutation of the best chromosomes $x_i^{'}$.

Where B_{Fc} is the best fitness value in the current population, B_{Fp} is the best fitness value in the previous population, ε is the maximum number of generations, and Θ is the minimum fitness threshold.

4. Time Series Forecasting Examples

To prove the forecasting efficiency of the proposed method that produces future values based on historical information; mackey-glass time series has been considered as a benchmark to compare the ability of generalization different methods due to its chaotic nature. Mackey-Glass is an artificial series widely used in the field of time series forecasting. It has special interesting characteristics. It is a series chaotic, which is defined by differential delay equation. RBFN which uses GAs to optimize its parameters was used to forecast points of the time series Mackey-Glass in short term case generated with the following expression:

$$\frac{ds(t)}{dt} = \alpha \cdot \frac{s(t-\tau)}{1+s^{10}(t-\tau)} - \beta s(t) \qquad (14)$$

where *x(t)* is the value of the time series at time *t*. The time series was constructed with parameter values and $\beta = 0.1$. Here, initial conditions used in our test bench are set as $s(0) = 1.2$ and $s(t) = 0$ when $t < 0$, doing $\tau = 17$. The total number of samples used in this experiment is 1200 of the mackey-glass time series as shown in Figure 4. The training is performed on 600 samples Figure 5 and the other 600 samples are used for testing the generalization ability of the proposed approach Figure 6. The proposed approach is simulated using MATLAB 7.1 under Windows XP with i3-330M Processor, 4GB DDR memory. We attempt to forecast a short-term prediction of the proposed method.

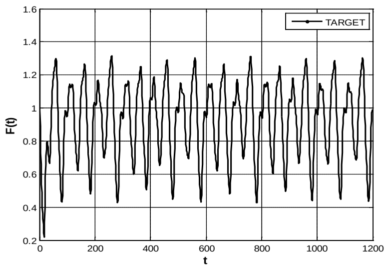

Figure 4. Mackey- Glass time series (1200 samples).

Parameters and initial value have been used as initial population generated using 250 chromosomes each chromosome divide in RBFs depends on the number of RBF in the hidden layer in each GA cycle. The number of RBFs (Neurons) increase depends on the output GA cycle and the stop condition. The crossover rate is 0.75 and the mutation rate is 0.075. The maximum generation numbers are 500.

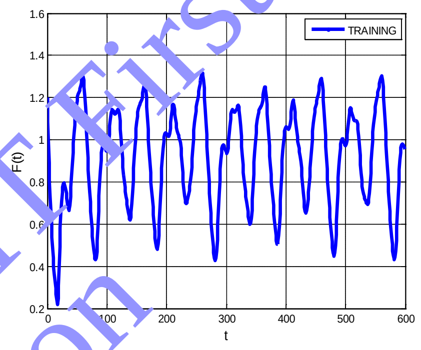

Figure 5. Mackey- Glass time series (600 training samples).

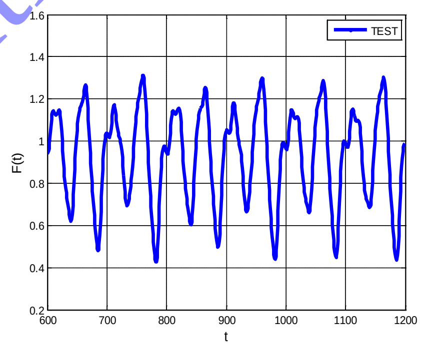

Figure 6. Mackey- Glass time series (600 testing samples).

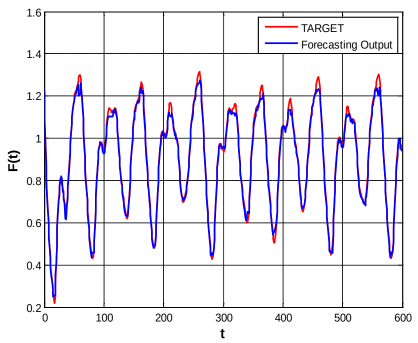

Figure 7. Forecasting with 20 RBFs.

Table 1. Forecasting Result of RMSE based on different methods step 6 (600 test data).

Approach	m	$RMSE_{test}$
Back-Prop NN	-	0.02
RBFNs Structure	12 RBF	0.003
Rojas. 2000. [28]	4 × 4 × 5 ×5	0.006
Gonzalez. 2001. [16]	16	0.004 ± 0.0002
Rivas. 2003. [26]	16	0.005 ± 0.0003
Awad. 2009. [3]	16	0.003 ± 0.0006
Awad. 2010. [2]	16	0.0014 ± 0.0004
Chandra. 2012. [10]	13	0.0094 ± 0.0005
Behmanesh. 2014. [5]	20	0.0011
Chen. 2014. [11]	3 for each cell	0.0093
Abdollahzade. 2015. [1]	14	0.0017
Proposed Approach	4	0.0183 ± 0.0210
	7	0.0072 ± 0.0012
	10	0.0035 ± 0.0004
	13	0.0019 ± 0.0007
	16	0.0011± 0.0004

In Table 1, the RMSE results obtained for short-term Mackey Glass time series. As it can be observed in applying the Hybrid enhanced approach that combine GAs and RBFNs achieve better forecasting result. Among the results obtained by the proposed approach we note that there is an improvement in RMSE compare with other methods already mentioned, with less number of RBFs used, which mean less complexity of the RBFNs and less time of execution.

From Figure 7, it is clear that the RBFN model based on GAs can be more accurate in forecasting Mackey Glass time series. The forecasting data curve and the objective data curve are approximately fitting; there is a good degree of improvement in the forecasting accuracy, which approximately equal 1.2%.

It can be seen from the result analysing that the proposed approach, which depends on initializing the RBFN parameters in the initial population of GAs and optimize weights using OLS, avoid stuck in local minima, it converges easily and improves forecasting performance. Actual values and the forecast values of the 600 samples presented in Figure 7 and the forecasting Error is presented in Figure 8. Although at some data samples, the hybrid RBFN and GAs produce more reasonable forecasting than other traditional methods, although the Mackey Glass parameters have high sensitivity.

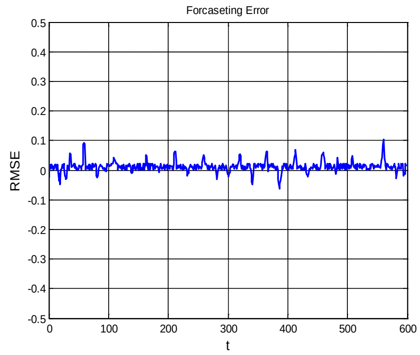

Figure 8. Forecasting RMSE with 20 RBFs.

The experiments demonstrate that only a small number of RBFs are needed for a given time series forecasting problem with better accuracy. This is because the GAs models provide more efficient center c position and radius value r. Moreover, the initial parameters of the GAs are not randomly generated, which make the parameter optimization process more effective, this produce best convergence and avoid the stuck in local minima. Predicting the Mackey Glass time series using the proposed hybrid method produce, can be seen as a process of decreasing the noise in a time series as shown in Figure 7.

5. Conclusions

In this paper, we proposed a hybrid approach depends on combining RBFNs and GAs used for forecasting the chaotic time series. For the purpose traditional GA has been modified to find the optimal topology and parameters of RBFN depends to given time series. This proposed approach optimizing the RBFs parameters using a modified GAs search strategy, which based on efficient technique of initialization the RBFNs parameters in the population chromosomes. The proposed approach used OLS to optimize the RBFN weights; the initialization of the centres depends on a classical algorithm of clustering and the width depends on k-nn algorithm. By using this process, RBFs in the population is cooperating to achieve a final solution and competing for survival, maintained to avoid stuck in local minimums, converge easily and improved forecasting performance. The behaviour of each basis function within the network is measured based on RMSE between the objective output and the real forecasting output. We have also concluded that the used of GAs process increment RBF in each GAs cycle, which means that we can find the minimum number of RBF that satisfied forecasting application, which means best RBFN topology for the specified forecasting problems. The proposed approach has been evaluated using short term Mackey Glass time series and the results obtained are comparable to those of the best and newest methods used for the same purpose

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