MIXED CONVECTION AND FLOW-REVERSAL IN A DIFFERENTIALLY AND ASYMMETRICALLY HEATED VERTICAL CHANNEL: APPLICATION TO CONVECTION COOLING OF CIRCUIT BOARDS

F. MECHIGHEL[1], M. KADJA[2], S. BEN AOUA[1] AND M. LAOUICI[1],

[1]Laboratoire LR3MI, Département de Génie Mécanique, faculté des sciences de l'ingéniorat, BP 12, Université d'Annaba (UBMA), 23000 – Algérie, E-mail farid.mechighel@etu.unilim.fr
[2]Laboratoire LEAP, Département de Génie Mécanique, Université de Constantine 1, 25000 – Algérie

ABSTRACT

A mathematical model for mixed convection heat transfer in a vertical channel with asymmetric wall temperatures counting situations of flow-reversal was presented. The model is firstly validated by considering numerical simulations of some test cases problem of mixed-convection available in the literature and then adapted to handle the problem of convection cooling of circuit boards.

Keywords: *Mixed Convection, Heat Transfer, Convective flow, flow-reversal*

NOMENCLATURE

Symbols:		**Dimensionless numbers:**	
L	channel length, m	Gr	Grashof number
b	channel width, m	Pe	Péclet number
k	thermal conductivity, $W \cdot m^{-1} \cdot K^{-1}$	Pr	Prandtl number
p	pressure, $N\ m^{-2}$	Re	Reynolds number
P	dimensionless pressure	Ri	Richardson number
r	dimensionless temperature range	**Greek letters:**	
T	temperature, K	α	thermal diffusivity, m2/s
u, v	velocity components, m/s	Θ	dimensionless temperature
U, V	dimensionless velocities	ρ	density, $kg\ m^{-3}$
x, y	Cartesian coordinates, m	ν	kinematics viscosity, m^2/s
X, Y	dimensionless coordinates	ΔT	temperature scale, K

1. INTRODUCTION

Convection heat transfer can be free (natural), forced or mixed in nature. When the fluid motion is caused by a density variation, due to a temperature difference, the situation is called *free* convection. When the fluid motion is caused by an external force, the convection mode is defined as one of *forced* convections. A state of *mixed* convection is the one in which both the natural and the forced convections are presented.

Mixed-convection heat transfer is important in several technical and scientific domains due to its common occurrence in engineering, industrial, and natural surroundings. For instance, heat transfer involving flow through a *parallel plate channel* is found in conventional *flat plate type solar collectors* and *equipment for electronic cooling*. In the above application, electronic components are usually mounted on parallel *circuit* *boards*, which are in general placed vertically in a cabinet and form *vertical parallel channels* through which *coolant* are passed. The coolant (usually air) can be propelled by free convection, forced convection, or mixed convection, depending on the power density of the circuit boards [1].

In the literature, mixed convection in a vertical plane channel has been widely studied by numerical methods [1-3] and experimental methods [4, 5]. In these studies, the *flow-reversal* (*reversed-flow*) was commonly identified. The present investigation deals with a vertical plate channel with asymmetric wall temperatures. A mathematical model for the problem of mixed convection was carried out.

2. MATHEMATICAL MODELING

The physical model considered in the present study is shown in figure 1, where uniform inlet conditions are supposed. The inlet velocity is a given quantity and it is assumed that in the physical situation, a fan is adjusted to maintain the given velocity when flow rate is modulated by mixed convection.

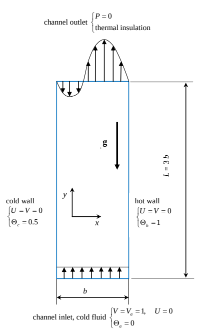


Figure 1: The physical problem.

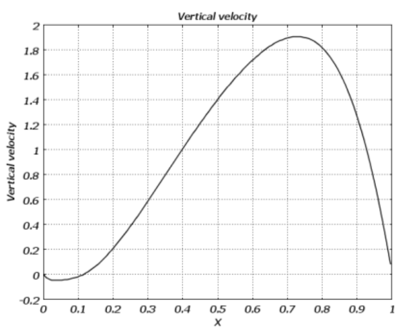


Figure 2. Vertical velocity profile at the channel outlet (present model).

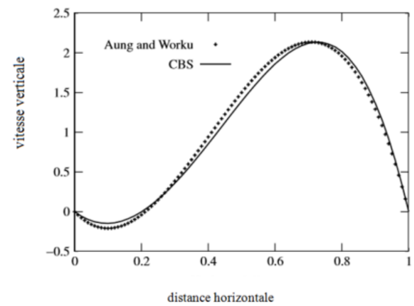


Figure 3. Results predicted in the references [2] and [6].

2.1 The governing equations

The non-dimensional Navier-Stokes and energy equations and the boundary conditions that describe the physical situation as shown in Fig. 1 are written in the Cartesian coordinate system as follows:

$$\partial U/\partial X + \partial V/\partial Y = 0 \tag{1}$$

$$\frac{\partial U}{\partial \tau} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y}\right) \tag{2}$$

$$\frac{\partial V}{\partial \tau} + U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + \frac{\mathrm{Gr}}{\mathrm{Re}^2}\Theta \tag{3}$$

$$\frac{\partial \Theta}{\partial \tau} + U\frac{\partial \Theta}{\partial X} + V\frac{\partial \Theta}{\partial Y} = \frac{1}{\mathrm{Re\,Pr}}\left(\frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y^2}\right) = \frac{1}{\mathrm{Pe}}\left(\frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y^2}\right) \tag{4}$$

Bottom ($Y = 0$, $0 \le X \le 1$) (inlet) $U = 0$, $V = 1$, $\Theta = \Theta_a = \Theta_0 = 0$ (5)

Top ($Y = Y_{\max} = L/b$, $0 \le X \le 1$) (outlet) $P = 0$, insulated (6)

Left plate ($X = b/b = 1$, $0 \le Y \le Y_{\max}$) (cold wall) $U = 0$, $V = 0$, $\Theta = \Theta_c = r_\Theta$ (7)

Right plate ($X = 0$, $0 \le Y \le Y_{\max}$) (hot wall) $U = 0$, $V = 0$, $\Theta = \Theta_h = 1$ (8)

where $X = x/b$, $Y = y/b$, $\tau = t v_0/b$, $U = u/v_0$, $V = v/v_0$, $P = p/\left(\rho v_0^2\right)$, $\Theta = \left(T - T_a\right)/\left(T_w - T_a\right)$ (with $T_w = T_h$ for the hot wall and $T_a = T_0$ for the channel inlet). The Boussinesq approximation for the buoyancy force term has been used and the dimensionless numbers are given as $\mathrm{Re} = v_0\, b/\nu$, $\mathrm{Gr} = g\,\beta \Delta T\, b^3/\nu^2$ and $\mathrm{Pr} = \nu/\alpha$, respectively. When Gr is positive, it is a buoyancy aiding flow; otherwise, it is an opposing flow. The term of the volume force in the case of the mixed convection problem is $\left(\mathrm{Gr}/\mathrm{Re}^2\right)\Theta$.

3. RESULTS AND DISCUSSION

3.1 Validation of the model

First test problem: Mixed convection problem studied in the references [2] and [6]

Firstly we consider a simple mixed convection problem, in a vertical channel, as shown in figure 1. This problem has been studied in the references [2] and [6]. It consists of two vertical plates which serve as the walls of the channel: The first wall is at the higher temperature ($\Theta_h = 1$) and the temperature of the colder wall is ($\Theta_c = r_\Theta = 0.5$). The cold fluid (air) incoming the channel from the bottom has ($\Theta_a = \Theta_0 = 0$). A uniform vertical velocity, of unit, is imposed at the inlet ($V_a = V_0 = v/v_a = 1$). The gravity direction is assumed act in the negative sense of *Y*. At the inlet the Reynolds number is ($\mathrm{Re} = v_0\, b/\nu$ = 100) and the Grashof number is assumed equal to 25000, this gives ($\mathrm{Gr/Re} = 250$ and $\mathrm{Ri} = \mathrm{Gr}/\mathrm{Re}^2 = 2.5$). At the outlet, a value of zero pressure is imposed, and the length of the channel is three times of the width ($L/b = 3$). The present example represents an illustration of buoyancy-driven convection heat transfer, although the buoyancy force aids the fluid to move very quickly by creating an upward flow near the wall, driven by the density variations. It should be emphasized, however, that at a grand Richardson number the flow-reversal is possible in this kind of the problem, as schematically shown in Fig. 1. In general, it is possible, in certain practical applications, that the flow was forced from the upper part of the channel (in the sense of *y* negative). Such a flow situation is called *opposed-flux* in which the *buoyancy-driven flow* is in the opposite sense of the *forced flow*.
Steady-state mixed convection numerical results are shown in figure 2. The unstructured mesh, used in the predictions, is refined near the solid walls and a total number of 4560 elements is employed in the simulation. The flow is almost *fully developed* at a vertical distance of 2 from the inlet. The ratio (Gr/Re) is 250 and further increase in this ratio will lead to stronger flow-reversal pattern. Comparison of the fully developed velocity profile obtained by the present model (Fig. 2) with the analytical solutions in [2] and numerical solution in [6] (Fig. 3) shows a good agreement.

Second test problem: Mixed convection problem studied numerically in [1] and analytically in [3]

The second test case considered is the mixed-convection problem between vertical parallel plates with the parameters ($\mathrm{Pr} = 0.72$, $\mathrm{Re} = 1$, $r_\Theta = 0.5$ and $\mathrm{Gr}/\mathrm{Re} = -140$ (i.e. $\mathrm{Ri} = -140$)). This problem was studied numerically in the reference [1] and analytically in [3]. At these flow conditions, the numerical resolution of the perturbed Navier-Stokes equations exhibits the existence of separation bubble at the hot wall (see [3]).
The existence of separation bubble, located near the heated wall, is also identified by the numerical solution presented in the reference [1] as shown in the figures 4. Fig. 4a shows the numerical solution where a separation bubble is found near the right boundary (hot wall). This point is further illustrated in Fig.4b, which display the results using another stretched grid system. The present numerical results are shown in the figures 5.

Examination of the figures 5a and 5b shows that the previous tendencies were also predicted by the present model. Thus, the present results agree well with the numerical results presented in Fig. 4.

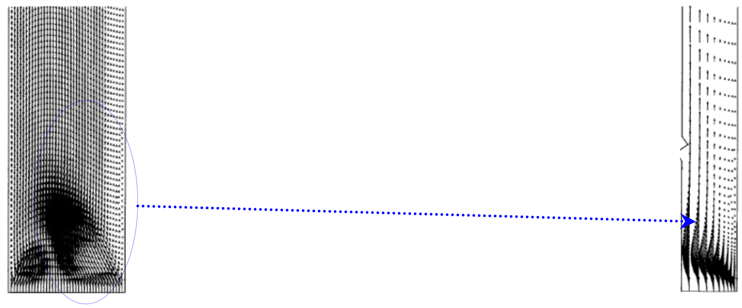


FIGURE 4A. Flow field obtained in the reference [1] (flow-reversal occurs at the hot wall) (Re = 1, Pr = 0.72, $r_{\Theta} = 0.5$, and Gr/Re = –140).

FIGURE 4B. Results from stretched grid 61 x 21 (flow-reversal occurs at the hot wall) (Re = 1, Pr = 0.72, $r_{\Theta} = 0.5$, and Gr/Re = –140) [1].

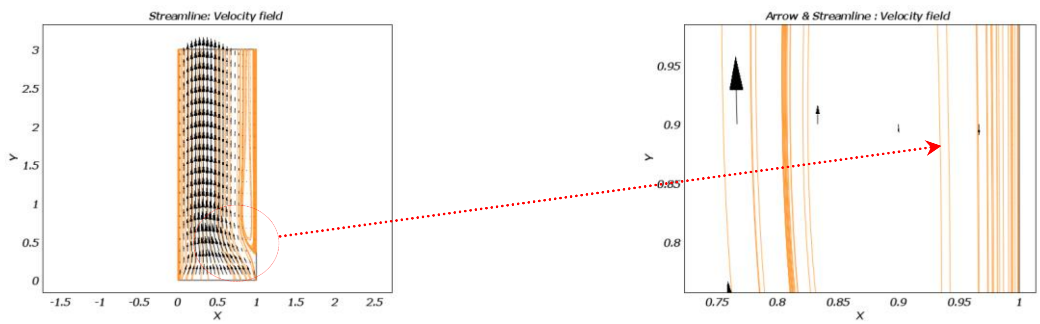


FIGURE 5A. Flow field obtained by the present model (flow reversal occurs near the hot wall).

FIGURE 5B. The zoom captured in the field clearly showing the flow-reversal near the hot wall.

3.2 Application of the model to the study of the process of convection cooling of circuit boards

The third studied example consists to model the air cooling of circuit boards occupied with multiple integrated circuits (ICs), which act as heat generations (sources) (figures 8 and 9). Generally the system is vertically or horizontally aligned boards with multiple in-line heat sources. Here we consider the vertically aligned boards (as shown in Fig.9a). This system uses generally natural or mixed convection for cooling. The forced convective flow is assured by a fan cooling (Fig.9a). The dimensions of the system are: (Board: the length (in the flow direction "*y*") is 0.13 m, and the thickness is 0.002 m (*x*-direction)), (ICs: the length and width (in *z*-direction) are both 0.02 m, and the thickness is 0.002 m), and the distance between the boards is 0.010 m. Both the boards and the ICs are in silicon (Si). It is expected that convective contributions caused by the induced (forced) flow of air dominate the cooling.

For simplicity in the present study we consider a simplified configuration (shown in Fig.9b) of the system. The previous model can easily extended to the study of this system. Note that in this case we have the presence of (one board and four ICs (solids)) and the air (fluid). In the fluid zone, the momentum and the heat transports are governed by the previous governing equations (1-4). While, in the solids (board/ICs) only the heat transport (conduction) is considered, which is assumed as steady-state. The governing equation for heat transport in the solids is given as

$$0 = k_{\mathrm{Si}}\left(\partial^2 T_s / \partial x^2 + \partial^2 T_s / \partial y^2\right) + Q_{\mathrm{SOURCES}} \qquad (9)$$

where T_s is the temperature in the solids, k_{Si} and Q_{SOURCES} are the thermal conductivity of silicon and the heat sources, respectively. The above equation should be written in dimensionless form following the previous procedure. For the boundary condition, we assume that the system (channel) wall is made of two parallel plates, with one (left) wall heated uniformly (from the source Q_{SOURCES}) and the opposite (right) wall insulated. A uniform air flow is made to enter the channel from the bottom (by means of the fan). Unfortunately for lack of space we cannot give further details.

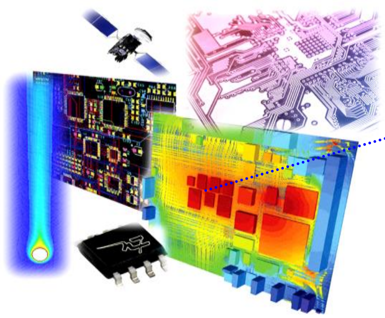


FIGURE 8. Actual view of a board with multiple in-line heat sources.

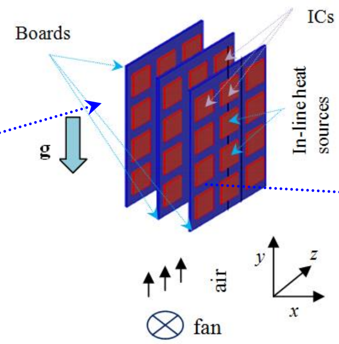


FIGURE 9A. A schematic representation of vertically aligned circuit boards with multiple in-line heat sources.

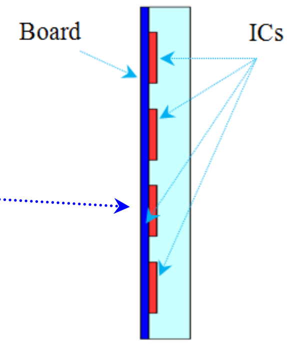


FIGURE 9B. The simplified configuration (2-D model).

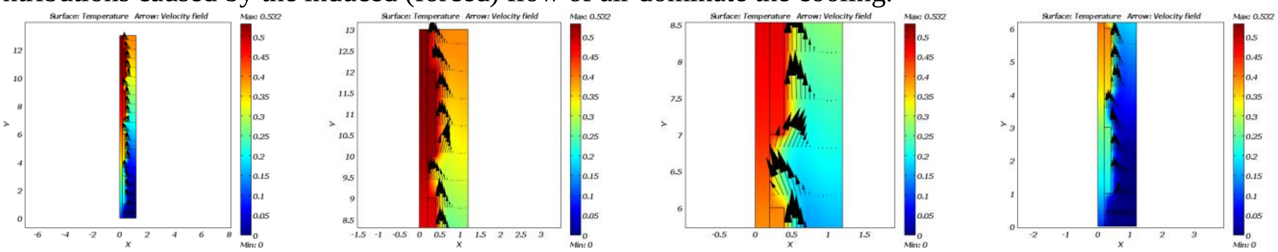
The heated circuit board vertical channel (depicted in Fig.9b) is studied numerically for situations where the buoyancy parameter $\text{Ri} = \text{Gr}/\text{Re}^2$ is relatively large. Numerical predictions have been achieved for various flow conditions as resumed in table 1. The flow-reversal is identified, which occurs initially near the channel outlet for the case when Ri is greater than a *threshold value*. The cold flow-reversal penetrates the channel from the outside and forms a *V-shaped recirculating flow* region in the downstream part of the channel. This region gradually spreads upstream as the buoyancy parameter (Ri) increases. The *counter-flow* motion, leading to *mixing* between the heated buoyant fluid and the V-shaped recirculation, is shown to be highly unstable and characterized by generation of *eddies* and *vortices* when the value of (Ri) is large. An increase in Re has the effect of pushing the reversed-flow downstream and making the recirculating region wider. Temperature fluctuations are predicted to provide insight into the complex phenomena being studied. The penetration depth of the flow-reversal, the local and average Nusselt numbers are also predicted and not presented (for lack of space). As an example of illustration, we have presented the results obtained for the flow conditions at (Re=50 and Gr =25000 which give Ri = 100). Flow and temperature fields are presented in figures 10 and 11. The previous tendencies may be observed (Fig.11).

TABLE 1. Some mixed convection flow conditions investigated

Gr	Re	Ri	**Convection mode**	**Reversed-flow**
0	1	0	Weak forced convection	no
	100	0	Strong forced convection	no
25000	10	250	Mixed convection	possible
	100	2.5	Mixed convection	not significant
	158	1.0	Mixed convection	not significant
	182	0.76	Forced convection dominant	no
250000	50	100	Mixed convection	yes
	100	25	Mixed convection	yes
	158	10	Mixed convection	yes and weak

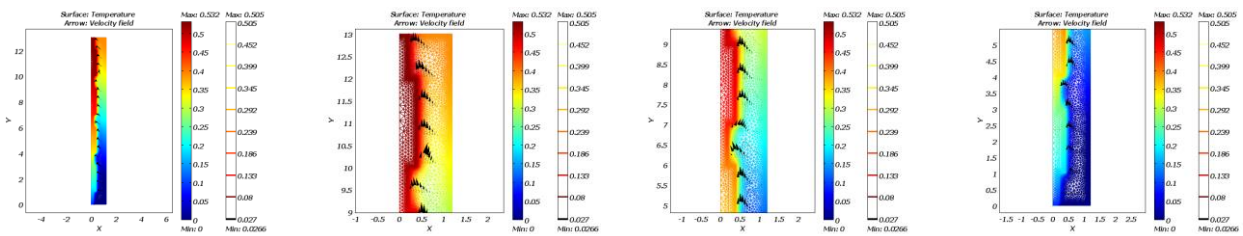
4. CONCLUSIONS

Mixed convection between vertical parallel flat plates with and without flow reversal has been inspected over the range of $\mathrm{Re} = [1 - 1000]$. A comparison between the present numerical solutions and those in the literature indicates that the present model can correctly evaluate the heat transfer on the heated wall. The model has been extended for the study of the implied convection cooling of circuit board system. It is found that convective contributions caused by the induced (forced) flow of air dominate the cooling.



(a) (b) (c) (d)

FIGURE 10. Predicted flow field: (a) in the whole solids/fluid system, (b) zoom in the upper region, (c) zoom in the central region and (d) zoom in lower region.



(a) (b) (c) (d)

FIGURE 11. Temperature field: (a) in the whole system, (b) upper region, (c) centre and (d) lower region.

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