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DETERMINING THE PREVENTIVE REPLACEMENT PERIOD BASED ON THE AGE OF SPARE PART

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ABSTRACT

The goal of this work is determining the periodicity which the preventive maintenance will execute for minimizing the operation cost. This type of preventive change is very used in practice by industrial manufacturing. In this study, we choose the replacement model based on age: the age of each part is known and the part is changed as soon as its age reaches the T_0 value. The mathematical model used is based on the Weibull law with Gamma =0. The results obtained are discussed according to the values of the parameters of the Weibull law Beta and Eta and of the cost of preventive maintenance and the cost failure, for which the minimal cost has been determined in each case.

Keywords: *Optimal Periodicity, Preventive Maintenance, Ratio Costs, Failure, Weibull Law, Replacement Based On Age.*

NOMENCLATURE

Symbols:

C_f	Cost of system failure, DA
C_p	Cost of system preventive replacement, DA
C_r	Cost ratios, $=C_p/C_f$
$C(T)$	Total cost per unit time, DA/h
CM	Corrective maintenance
$f(t)$	Failure density of the system, $=R'(t)$
$F(t)$	Unreliability of the system er time t, $=1-R(t)$;
$R(t)$	Reliability of the system over time t
MTBF	Mean time between failure
PM	Preventive maintenance
t	Time, h
T	Replacement time, h
T_0	Periodicity optimal of replacement, h

Greek letters:

β	Weibull distribution shape parameter
ε	Fraction of time, $\varepsilon << T_0$, h
γ	Weibull distribution location parameter, h
η	Weibull distribution scale parameter, h
$\lambda(t)$	failure rate;
μ	Variable, $=(t/\eta)^\beta$
$y(\mu)$	Function of cost ratio

1. INTRODUCTION

The complexity of the phenomena of failures leads us to seek means of improving the strategies and the policies of maintenance to make it possible the equipment to adequately fulfill the functions for which it was conceived. The most important problem in the mathematical methods of maintenance is to conceive maintenance planned with two options of maintenance: preventive replacement and corrective replacement. For these reasons, an important area of reliability theory is the study of various maintenance policies that seek the way to reduce operating costs and the risk of a catastrophic failure. So, Al-Najjar [1] showed that maintenance expenses vary depending on the type of industry; figures typically encountered are in the order of $15-40\,\%$ of production costs. Therefore, it is necessary to pay more attention to this important subject area. Furthermore, timely preventive maintenance (or replacement) is also beneficial to support normal and continuous system operation. Therefore, it becomes desirable to determine an optimal replacement policy for the system. Barlow and Proschan [2] were proposed an age-replacement policy, where an operating unit is replaced at time of failure, or at age *T*, whichever comes first. Nosoohi and Hejazi [5] have presented a novel multi-objective model for preventive replacement of a part over a planning horizon. The proposed model considers different objectives and practical issues, such as corrective replacement and its consequences, residual lifetime objective, and somehow productivity index.

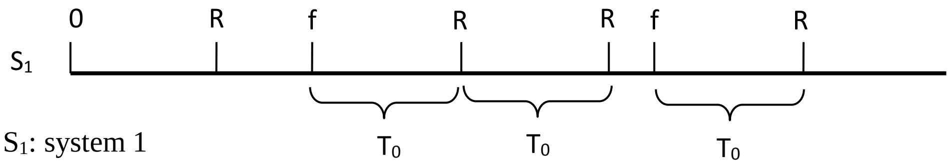
Halim and Tang [6] have extensively studied the replacement problems of deteriorating systems. Typically, the time between failures is characterized by lifetime distribution in which parameters are estimated from historical data. Jung and Park [7] have developed the optimal periodic preventive maintenance policies following the expiration of warranty. They have considered two types of warranty policies to discuss such optimum maintenance policies: renewing warranty and non-renewing warranty. From the user's perspective, the product is maintained free of charge or with prorated cost on failure during the warranty period.

Chien and Chen [8] have presented a spare ordering policy for preventive replacement with age-dependent minimal repair and salvage value consideration. The spare unit for replacement is available only by order and the lead-time for delivering the spare due to regular or expedited ordering follows general distributions. Barlow and Hunter [9] have proposed two mathematical models for the determination of the policy of optimal replacement minimizing the cost operation of the production system. These models are called Block Replacement Models and Age Replacement Models**.** The main property for the block replacement is that it is easier to administer in general, since the preventive replacement time is programmed to the in advanced and we do not need the watch of the system age. In the age replacement model, as it is well recognized, if the unit doesn't fail until a prespecified time, then it is replaced by a new one preventively; otherwise, it is replaced at the failure time. This model plays a central role in all replacement models, since this has been proved by Bergman [10] if the replacement by a new unit is the only maintenance option.

Our objective is to determine the most appropriate period, from an economic point of view, to make replacements of mechanical parts. We take in consideration all the parameters that involved, so that, this operation can be profitable. We propose an analytical and numerical method for solving the resulting differential equation and we give some numerical examples. This work aims at studying models of replacements based on the age: the age of each part is known and the part is changed as soon as its age reaches the T_0 value.

2. MATHEMATICAL MODEL

This study consists in making a preventive replacement when the equipment reached the T_0 age is the period of preventive replacement selected (figure 1). The duration of the T_0 period was given in order to carry out a preventive replacement a little before the moment or we estimate that the equipment is likely to break down. That makes it possible to minimize the costs. However if a failure occurs, the faulty equipment is replaced by nine. The generation of the moments of failures is made by a random function. One of the functions often used is the Weibull law distribution. The latter is interesting taking into account its flexibility and the great number of laws which it can simply cover by varying parameters (normal law, exponential law, etc).



S_1: system 1
R: replacement
f: failure
T_0: spare part age

FIGURE 1. Replacement when the spare part age T_0 is reached.

The differences compared to the block replacement model case [10] are the following:

- The number of parts to be changed is reduced because we aren't likely to change a part which has just been failing and replaced.
- It is necessary to know the age of each part, which requires a special organization.
- The preventive exchanges are more expensive because they relate only to one part each time.

The value T_0 which corresponds to the periodicity of preventive maintenance is that which minimizes $C(T)$ given by the following expression:

$$C(T)=\frac{C_f\left(1-R(T_0)\right)+C_P.R(T_0)}{\int_0^{T_0}R(t)dt}\,;T_0\geq 0 \tag{1}$$

Where $C(T)$ the average total cost by part and time unit.

The problem is, of course, to derive the optimal block replacement time T_0 that minimizes $C(T)$.

$$C^{'}(T)=0 \quad\Leftrightarrow\quad \left(f(T_0)\int_0^{T_0}R(t)dt\right)\left(C_f-C_P\right)R(T_0)\left[C_f-R(T_0)\left(C_f-C_P\right)\right]=0 \tag{2}$$

$$C_f\,R(T_0)=\left(C_f-C_P\right)\left[f(T_0)\int_0^{T_0}R(t)dt+R(T_0)^2\right] \tag{3}$$

$$\frac{1}{1-C_P/C_f}=\frac{f(T_0)\int_0^{T_0}R(t)dt+R(T_0)^2}{R(T_0)} \tag{4}$$

We put: $C_P/C_f=C_r$; then we obtain: $\frac{1}{1-C_r}=\lambda(t)\int_0^{T_0}R(t)dt+R(T_0)$ (5)

We consider the Weibull law with three parameters: γ, β and η which aptly describe the behavior of the material studied. Then, in the case of Weibull law with $\gamma=0$: this means that the origin of time is taken equal at zero, and the equipment was operated at *t = 0*. It's the most common case in practice. So we have:

$$R(T)=e^{-(t/\eta)^{\beta}} \tag{6}$$

And equation (5) becomes: $\frac{1}{1-C_r}=\frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1}\int_0^t e^{-\left(\frac{t}{\eta}\right)^{\beta}}dt+e^{-\left(\frac{t}{\eta}\right)^{\beta}}$ (7)

We put: $y(t)=\frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1}\int_0^t e^{-\left(\frac{t}{\eta}\right)^{\beta}}dt+e^{-\left(\frac{t}{\eta}\right)^{\beta}}=\frac{1}{1-C_r}$ (8)

$$y'(t)=\beta\left(\beta-1/\eta^{2}\right)(t/\eta)^{\beta-2}\int_{0}^{t}e^{-\left(\frac{t}{\eta}\right)^{\beta}}dt \qquad (9)$$

2. ANALYTICAL SOLUTION

The analytical study of this equation shows:

- For $0<\beta<1: y'(t)<0; \forall t>0$ and $0<y(t)<1$. The equation (8) doesn't have a solution because $C_r \notin]0,1[$. Moreover, this case has no practical interest, since the material is in youth period.
- For $\beta=1$: $y(t)=1$. Equation (8) is equivalent to $C_r=0$. This preventive maintenance doesn't have interest.
- For $\beta>1$: $y'(t)>0 ; \forall t>0$. Equation (8) has a single solution for $0<C_r<1$.

Let us check that the solution of the equation corresponds to a minimum at the cost. Then, we study the limit of: $\lim C(t)$ when $t\to 0$ and $\lim C(t)$ when $t\to +\infty$.

$$\lim_{t\to 0} C(t)=+\infty \qquad (10)$$

$$\lim_{t\to +\infty} C(t)=\frac{C_f}{\eta\Gamma\left(1+1/\beta\right)} \qquad (11)$$

In this case, average cost $C(t)$ has the form which is represented on figure 2. Thus, in the case of a material of wear (β>1), this type of preventive maintenance has an interest $\forall C_r \in]0,1[$.

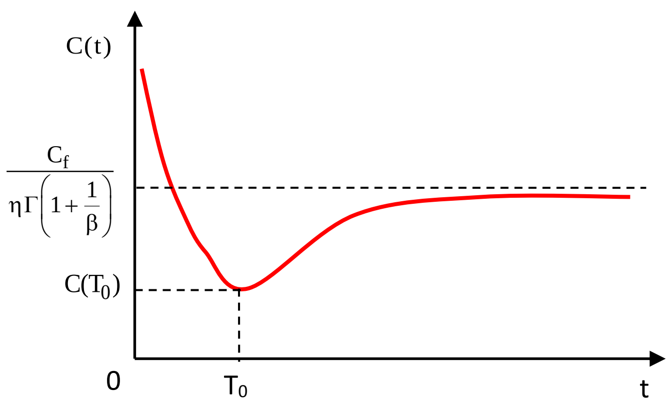


FIGURE 2. Curve of in $C_m(t)$ the case of $\beta>1$.

3. NUMERICAL SOLUTION

By observing the condition of $0<C_r<1$ and with β>1, we varied the costs ratio from 0.25 to 0.75, i.e. C_r = 0.25; 0.5 and 0.75, corresponding to C_f= 40000 and C_p=10000; 20000 and 30000, respectively.

The observation of figure 3 representing the average total cost per unit of time and part C according to the time of preventive replacement T if η=2000 and C_r=0.25 for various values of the parameter of form β watch that around the T_0 optimum the cost seems to vary very little. In this case, we note the presence of a minimal value of C corresponding to the period most adapted to carry out the operation of MP. This cost is appreciably depending on the value of the form parameter β. It varies from 5.64 for β=6.9 to 14.66 for β=2. Figure 4 represents the variation of the average total cost per unit of time and part C according to time if η=2000 and C_r=0.5 for various values of the parameter of form β. For β =2, we note that the period of MP starts beyond T_0=947 where C_m is minimal (equal to 22) and it will constitute to be it as from this moment until T_0=1890 where C_m is minimal (equal to 21). Whereas the optimal periodicity corresponding to the minimal value of C (C_m=20.62) is equal to T_0=1459. While, for the other values of β, the value of the minimal cost C_m corresponding to the period of MP varies from 11.09 for β=6.9 to 18.02 for β= 2.5, and the period of optimal replacement correspondent varies from 1076 for β=6.9 to 1208 for β=2.5. In a similar way, it is noted that C_m and T_0 proportionally vary the ones compared to the others according to the values of β. For example, in the case of β =3, C_m = 15.97 and T_0=1125. Whereas for in the case of β =6, C_m = 11.48 and T_0=1071.

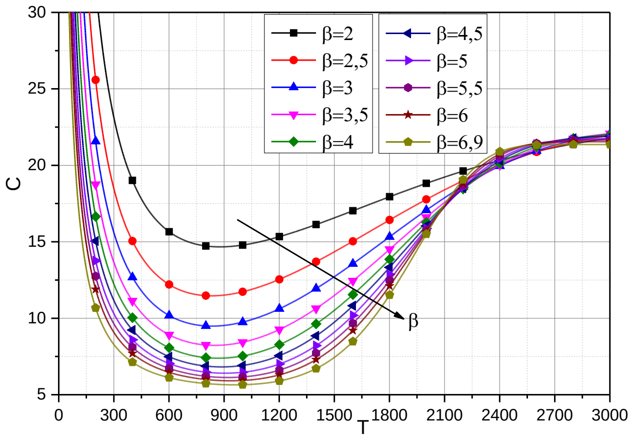


FIGURE 3. Variation of C according to time for various values of β : $\eta = 2000$, $C_r = 0.25$.

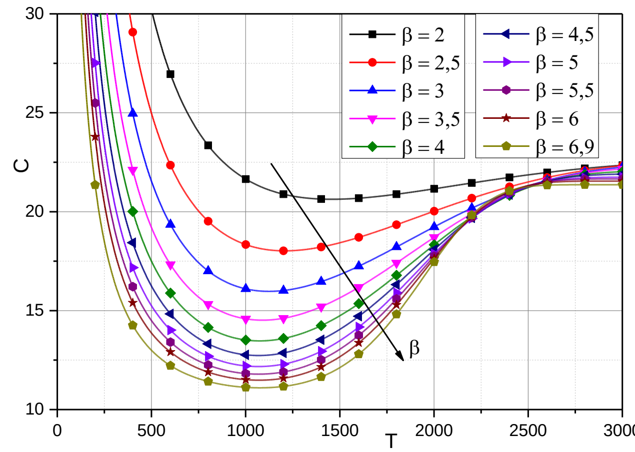


FIGURE 4. Variation of C according to time for various values of β : $\eta = 2000$, $C_r = 0.5$.

According to figure 5, which represents the variation of the average total cost per unit of time and part C according to time if η=2000 and C_r=0.75 for various values of the parameter of form β, we note a increase proportional of C_m and T_0 for all the values taken by β. This obviously little is explained by increased C_f, which implies more action of corrective maintenance what this reflects on the increase in the optimal periodicity replacement T_0 in preventive. Thus, for β=2, we note that the period of MP starts beyond T_0≥2500 where C_m is minimal (equal to 22.61) and it will constitute to be it as from this moment. While, for the other values of β, the value of the minimal cost C_m corresponding to the period of MP varies from 16.43 for β=6.9 to 22.25 for β= 2.5, and the period of optimal replacement correspondent varies from 1203 for β=6.9 to 11852 for β=2.5. In a similar way, it is noted that C_m and T_0 proportionally vary the ones compared to the others according to the values of β. For example, in the case of β =3.5, Cm = 19.75 and T_0=1399. Whereas for in the case of β =6, Cm = 16.86 and T_0=1221.

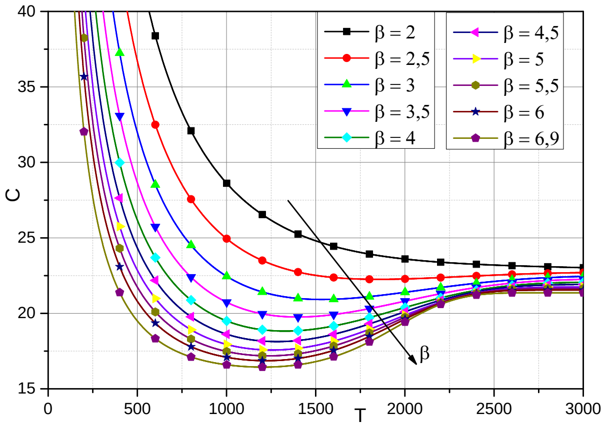


Figure 5. Variation of C according to time for various values of β : $\eta = 2000$, $C_r = 0.75$.

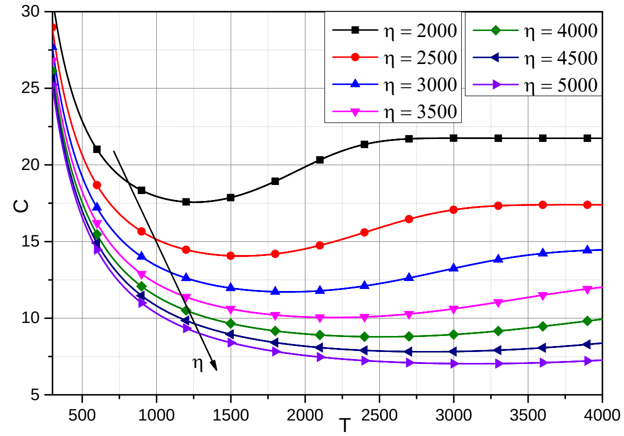


Figure 6. Variation of C according to time for various values of η : $\beta = 5$, $C_r = 0.75$.

Since the values taken by the scale parameter η, we must expect that T_0 evolves in a way proportional with η. Thus, figure 6 represents the variation of the total cost average C according to time in the case of β=5, C_r=0.75 and for various values of the parameter of scale η. Noting us that C decreases with the increase in the values of η, whereas the optimal period of T_0 replacement increases. Thus, for η=2000; C_m=17.57 and T_0= 1254, while for η =5000; C_m=7,02 and T_0=3135. Knowing that the scale parameter η represents an approximate value of MTBF, this evolution of C is completely logical. Indeed, for a type of spre part having a relatively weak MTBF, its duration of exploitation is also small and C_m is relatively high. Whereas for a spare part which has a relatively high MTBF, its duration of exploitation is large and C is relatively weaker.

5. CONCLUSION

Determining the optimal periodicity for the preventive replacement might be obtained by the replacement model based on age. The standard is to calculate the average total cost per time unit and per item, to get the minimum period corresponding to this minimum as an optimal time to perform the preventive maintenance. This cost comprises the cost of preventive maintenance and the cost of biased probability for the failure of corrective maintenance. An analytical study that has been carried out in the case of a Weibull distribution and the resulting differential equation has been solved under certain mathematical conditions. After that, this equation has been numerically solved for the different parameters of this problem which are the cost ratio of maintenance, the shape parameter and the scale parameter. The results were analyzed and discussed. Their applications to real cases can provide to maintenance service a key element in choosing the most suitable time to perform preventive maintenance at minimum cost.

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