

# MATHEMATICAL MODELING AND ENERGY ESTIMATION OF ANAEROBIC BIOREACTORS

Abdelouahab ZAATRI<sup>1</sup>, Norelhouda AZZIZI<sup>2</sup> and Ridha KELAIAIA<sup>3</sup>

<sup>1</sup>Département de Génie Mécanique, Faculté des Sciences de l'ingénieur, azaatri@yahoo.com

<sup>2</sup>Département de Mathématiques, Faculté des sciences exactes, azzizinorelhouda@yahoo.fr  
Laboratoire des Applications de technologie Avancée (LATA)  
Université de Constantine 1, Constantine, Algérie

<sup>3</sup>Département de Génie Mécanique, Faculté des Sciences de l'ingénieur  
Université 20Aout1955, Skikda, Algérie, ridakelaia@hotmail.com

## ABSTRACT

This work presents the study, the numerical simulation, and some experimental analysis of anaerobic bioreactors as well as the estimation of the produced annual energy.

A mathematical model corresponding to biotechnological processes of two-phase anaerobic digestion has been implemented to simulate the bioreactor operation. Simulation results allow understanding the degradation of the substrate, the growth of bacteria, and the production of methane.

On the other hand, to size a bioreactor, it is necessary to estimate its produced annual energy. To this purpose, we consider a completely mixed continuous anaerobic digester in steady state operation according to Chen-Hashimoto model. This model enables to link the annual produced energy with respect to the bioreactor characteristics and to the input products.

**Key words:** *mathematical modeling, Energy estimating of bioreactors, Model of Chenand-Hashimoto, bioreactors, Anaerobic Digestion.*

---

## 1. INTRODUCTION

Mathematical modeling and simulation are flexible and economical means for analyzing the operation of bioreactors and predicting their performance. The first mathematical models of anaerobic bioreactors have been proposed in the 1970s [1, 2]. Depending on the number of biochemical processes considered, there are more or less complex models that have been proposed. The ADM1 model (Anaerobic Digestion Model No. 1) is a model that has been developed by researchers at the IWA (International Water Association) [3]. It is the most complete model to simulate the anaerobic reactors. However, this model is very complex because it is very detailed. It requires more than 80 parameters to be tuned. A more practical mathematical model named AM2 which is much less complex corresponds to the biological process of anaerobic digestion in two phases. This last model is the one we have used for our simulation.

The work presented in this paper concerns the design, simulation and experimental realization of anaerobic fed-batch digesters intended for the production of methane. The AM2 model has been implemented to simulate the methane production in the bioreactors. Some model parameters were estimated by an extensive literature review, while others were evaluated in order to obtain consistent results qualitatively and quantitatively.

An experimental bioreactor with a capacity of 200 liters was achieved. The substrate was provided from the biological wastewater treatment plant in the region of Constantine.

The annual energy produced was estimated by means of Chen and Hashimoto model that applies to continuous digesters. This model enables in one hand, to estimate the annual energy production of

methane being given the volume of the bioreactor and the organic matter. On the other hand, it enables to sizing the bioreactor given the amount of the expected produced energy.

## 2.EQUATIONS OF THE DYNAMICAL MODEL

The mathematical model, based on the laws of grow, is the AM2 model. It involves the following dynamic variables:  $X_1$  and  $X_2$ , are respectively the concentration of the acidogenic bacteria population and the concentration of the methanogenic bacterial population while  $S_1$  and  $S_2$  are respectively the concentration of the substrate of carbonaceous material and the substrate concentration of volatile fatty acids.

For a batch system, the mathematical model is expressed in the form of acoupled differential equations of the first order system:

$$\frac{dX_1}{dt} = \mu_1 * X_1 ;$$

$$\frac{dX_2}{dt} = \mu_2 * X_2$$

$$\frac{dS_1}{dt} = -k_1 * \mu_1 * X_1 ;$$

$$\frac{dS_2}{dt} = -k_2 * \mu_1 * X_1 - k_3 * \mu_2 * X_2$$

The flow of methane, which is the expected product, depends directly on the growth of methanogenic bacteria population  $X_2$  according to the relation:

$$Q_{ch_4} = k_4 \times \mu_2 \times X_2$$

We distinguish nine parameters involved in this model which are ( $\mu_{1max}$ ,  $K_{S1}$ ,  $\mu_{2max}$ ,  $K_{S2}$ ,  $K_{I2}$ ,  $k_1$ ,  $k_2$ ,  $k_3$ ,  $k_4$ ).

To solve this system of differential equations, we must also provide the initial conditions, which are the initial quantities of the concentrations of substrates and bacteria at the start of bioreactor:  $S_1(0)$ ,  $S_2(0)$ ,  $X_1(0)$  and  $X_2(0)$ .

## 3.RESULTS

The model AM2 has been implemented for simulating the operation of the anaerobic bioreactors. An extensive analysis of the bibliographic literature showed that very little research has provided an estimate of some of the parameters of this model. For parameters that have been provided by different authors, there is a strong dispersion even when it comes to the mechanization using waste water as substrate [1], [4-7].

In fact, outside the parameters of the maximum growth rate for the acidogenic and methanogenic bacteria, the literature provides almost no data on other parameters. In our simulation, the growth parameters were evaluated by the following values:

$$\mu_{1max} = 0.4 \text{ /day};$$

$$K_{S1} = 35 \text{ mg/l};$$

$$\mu_{2max} = 0.4 \text{ /jour};$$

$$K_{I2} = 170 \text{ mg/l}$$

$$K_{S2} = 4 \text{ mg/l};$$

Moreover, due to the lack of data for the parameters ( $k_1$ ,  $k_2$ ,  $k_3$ ), these parameters were estimated by trial and error trying to get simulation results qualitatively and quantitatively consistent [9-11]. The initial values were empirically estimated. The parameter values used are:

$$k_1 = 50; k_2 = 50; k_3 = 15; S_1(0) = 10 \text{ g/l}; S_2(0) = 2 \text{ g/l}; X_1(0) = 0.4 \text{ g/l} \text{ and } X_2(0) = 0.01 \text{ g/l}.$$

Note: the scale of the substrate  $S_1$  is halved to allow better visualization of the results

A simulation result to visualize graphically the temporal evolution of the substrates  $S_1$  and  $S_2$  as well as the concentrations of bacteria's  $X_1$  and  $X_2$  are shown in Figure 1. According to Figure 1, we notice that for the used values of the parameters, there is an exponential decreasing of the substrate  $S_1$  which is almost completely consumed in 15 days because of its decomposition by acidogenic bacteria  $X_1$ . Meanwhile, the substrate  $S_2$  begins to be generated during these 15 days; then follows its consumption by the methanogenic bacteria. This substrate will be almost completely decomposed into biogas over a period of about one month.

After the disappearing of the substrates over a period of about one month, the concentrations of acidogenic and methanogenic bacteria will stabilize at some constant values. We notice again that the mathematical model does not predict their future evolution.

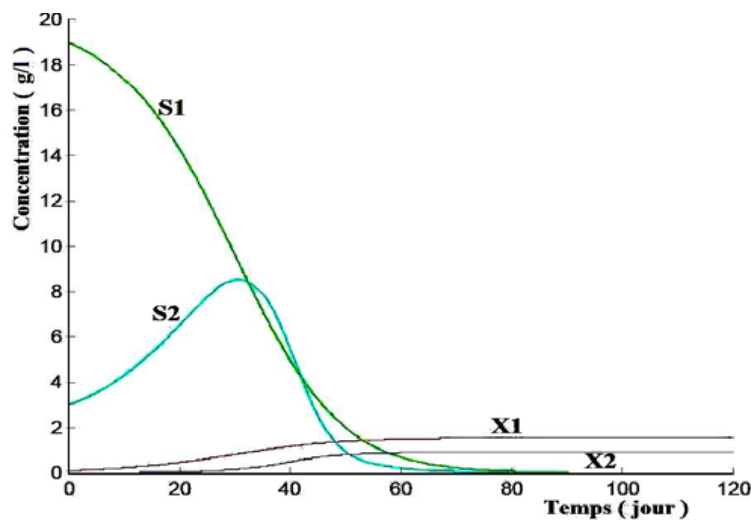


FIGURE 1. Temporal evolution of substrates and bacteria's

- To estimate the methane production, the literature provides data corresponding to  $k_4$  which is closer to the value  $75 \text{ l}^2/\text{mg}$  [1, 4]. According to this value of  $k_4$ , simulation results to visualize graphically the temporal evolution of methane flow  $Q(t)$  and the cumulated quantity of methane  $C(t)$  are shown in Figure 2 (the scale of  $C(t)$  is reduced 10 times).

Note that the curve  $Q(t)$  is presenting a rapid increase at the beginning of the launch of the bioreactor. It reaches a maximum in about 20 days and then begins to decline until almost disappearing at about 35 days. This type of behavior is expected from a batch digester.

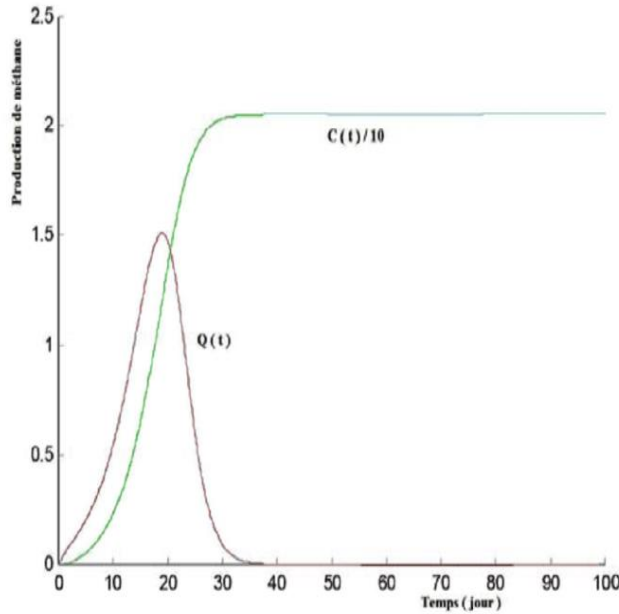


FIGURE 2. PRODUCTION OF METHANE

#### 4. ESTIMATION OF ANNUAL BIOGAS ENERGY

To estimate the power generated by a methane digester, we consider a completely mixed continuous anaerobic digester in steady state operation. We use the Chen-Hashimoto model which is expressed [15] as:

$$Q_m = VVJ * V(1)$$

Where:

- $Q_m$  : Production of methane ( $m^3/d$ )
- $V$ : Biodigester volume ( $m^3$ )
- $VVJ$ : Technological efficiency ( $m^3/m^3d$ ), which can be expressed as follows:

$$VVJ = B * (MO/HRT) \quad (2)$$

- $B$ : Biological efficiency.
- $MO$ =matter oxidizable.
- $HRT$ : hydraulic retention time( $HRT=Q/V$ ).

$$B (m^3CH_4 / kgMo) = B_0 \cdot [1 - (K/(\mu_m \cdot HRT) + K - 1)] \quad (3)$$

Where:

- $K$ : Constant of inhibition
- $\mu_m$ : kinetic coefficient.
- $B_0$ : Production potential of methane.

And

$$B = B_0 * [1 - (K/\mu_m - HRT)K - 1] \quad (4)$$

With

$$\mu_m = 0.013 * T - 0.129 \quad (5)$$

$$K = 0.6 + 0.021 \cdot 100.05 \cdot Mo \quad (6)$$

The final expression of methane production with respect to the different parameters that intervene in this model can be written as:

$$Q_m = B_0 [1 - (K/\mu_m - HRT + K - 1)] (MO/HRT) * V(7)$$

As example, let's consider a continuous bioreactor of volume  $V = 33 \text{ m}^3$ , with a volumetric flow rate  $Q=6 \text{ m}^3/\text{d}$ . The hydraulic retention time can be deduced as  $HRT = V/Q = 5.5 \text{ days}$ . In our regional context, we use the organic fraction of solid waste mixed with the activated sludge in mesophilic condition, with matter oxidizable  $MO = 6 \text{ l/kg}$ . At a temperature of  $25 \text{ }^\circ\text{C}$ , the kinetic coefficient can be estimated as

$$\mu_m = 0.013.T * 0.129$$

and  $B_0 = 0.10 \text{ m}^3/\text{KgMO}$ ; therefore the estimate energy produced by day is about  $Q_m = 0.2285 \text{ m}^3$ . If we consider a functioning of the reactor during 330 days per year, we can estimate the annual production of energy as  $E_{\text{bio}} = Q_m * 330 = 75.3943 \text{ m}^3/\text{year}$ .

This energy production can be converted into various other forms of energy such as heat, electricity and both. One  $\text{m}^3$  of methane provides an equivalent of heat energy of about 8570 kcal. It provides, when converted into electricity, about 10 kWh. If we convert the annual methane production into electricity, we can expect energy of about 754kWh/year.

## 5.CONCLUSION

We have presented a modeling and a simulation of the production of methane by using the AM2 model. The simulation results allow to understand the qualitative and quantitative experimental bioreactors. The experimental results are satisfactory and encouraging since we could actually produce methane despite the simplicity of the devices used. The estimate of energy is of paramount importance in the design of bio-anaerobic digesters. The Chen-Hashimoto model was used to estimate the annual methane production.

## REFERENCES

- [1] N. Noykova, T. Müller, M. Gyllenberg and J. Timmer, Quantitative Analysis of Anaerobic Waste Water Treatment Processes: Identifiability and Parameter Estimation, *Biotechnology and Bioengineering*. Vol. 78.N°1, 89 – 103, 2002.
- [2] U. Zaher, P. Pandey and S. Chen, A simple elemental continuity based model application to study the anaerobic microbial activity for the treatment of dairy manure, *Appl. Math. Modelling*, 33, 2009.
- [3] V. A. Vavilin, L. Y. Lokshina and S. V. Rytov. The <Methane> Simulation Model as the First Generic User-Friend Model of Anaerobic Digestion, *Vestnik Moskov skogo Universiteta*. Vol. 41, N°6, 22 – 26, 2000.
- [4] D. Simon. *Optimal State Estimation*. Wiley, 2006.
- [5] M. Gerber and R. Span, An Analysis of Available Mathematical Models for Anaerobic Digestion of Organic Substances for Production of Biogas, *International Gas Union Research Conferences*. IGRC, 2008.
- [6] D. T. Hill and C. L. Barth, A Dynamic Model for Simulation of Animal Waste Digestion, *Journal Water Pollution Control Federation*, Vol. 10, 2129 - 2143, 1977.
- [7] G. Lemon, R. J. King, H. M. Byrne, O. E. Jensen, and K. M. Shakesheff, Mathematical modelling of engineered tissue growth using a multiphase porous flow mixture theory, *Journal of Mathematical Biology*. Vol. 52, N°5, 571–594, 2006.

- [8] M. P. Bryant. Microbial Methane Production-Theoretical Aspect, *Journal of Animal Science*, Vol. 48, N°1, 193 – 201, 1979.
- [9] O. Bernard and J. L. Gouzé, Transient behavior of biological loop models with application to the droop model, *Mathematical Biosciences*, Vol. 127.N°1, 19-43, 1995.
- [10] D. Morau, S. Dumas, L. Adelard and J. C. Gatina, Optimization of the Anaerobic Digestion of Solid Waste by Addition of Leachate.  
[www.iswa.org/fileadmin/galleries/.../Presentations/Morau.pdf](http://www.iswa.org/fileadmin/galleries/.../Presentations/Morau.pdf).
- [11] R. Escudié, T. Conte, J. P. Steyer and J. P. Delgenès, Hydrodynamic and Biokinetic Models of an Anaerobic Fixed-Bed Reactor, *Process Biochemistry*.Vol. 40, N°7, 2311 – 2323, 2005.
- [12] T.G.Muller, N.Noykova, M.Gyllenberg and J. Timmer, Parameter Identification in Dynamical Models of Anaerobic Waste Water Treatment, *Mathematical Biosciences*. Vol. 177-178,147–160, 2002.
- [13] I.Simeonov, V.Lubenova and I.Queinec, Parameter and State Estimation of an Anaerobic Digestion of Organic Wastes Model with Addition of Stimulating Substances, *Bio-automation*, Vol. 12, 88– 105,2009.
- [14] X.Chen, R.T.Romano and R.Zhang, Anaerobic Digestion of Food Wastes for Biogas Production, *International Journal of Agricultural and Biological Engineering*, Vol. 3.N°4,51 – 62, 2010.
- [15] B.T. Nijaguna, *Biogas Technology*, New Delhi, 2009.
- [16] F. Vedrenne, Etude des processus de dégradation anaérobie et de production de méthane au cours du stockage des lisiers, *Thèse Science de l'environnement*, ENSA de Rennes, 2007.
- [17] A. Zaatri, N. KacemChaouche et M. Karaali, Etude de bioréacteurs anaérobies expérimentaux pour la production de méthane, *Revue des Energies Renouvelables*, Vol. 14, N°2, 291 – 300, 2011.
- [18] Y. R. Chen and A. Hashimoto, Kinetic methane fermentation, *Biotech.Bioenerg*, Symp.Vol. 8,209-281, 1979.